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FILM COOLING DURING INJECTION OF HETEROGENEOUS SUBSTANCE IN A TURBULENT BOUNDARY LAYER

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ABSTRACT. This paper deals with the effectiveness of film cooling of a heat insulating plane wall during local injection of a heterogeneous substance into the boundary layer. The turbulent boundary layer is formed on a solid wall behind the zone of injection. It is assumed that heat flow through the wall is zero. The cooling gas is injected through a tangential slit or through a porous section. If a liquid substance is used or if chemical reactions occur in the initial plate section, the effect of the products formed is considered to be the same as of a heterogeneous substance. The results are compared with those obtained by other authors.

Most works on film cooling are dedicated to investigation of the influence /103[1] of injection of a homogeneous gas. In [1], the case of tangential injection of gas into the boundary layer is investigated when its heat capacity c_{p_s} differs little from the heat capacity of the main stream c_{p_0}.

The purpose of the present work is an investigation of the effectiveness of film cooling of a heat insulated flat wall by local supply of a foreign material at the turbulent boundary layer.

1. If we ignore thermodiffusion and barodiffusion and diffusion heat conductivity, the equation for the energy in the boundary layer over a flat wall can be written in general form as follows [2]:

$$k = \frac{R_{\Delta x}}{R_s^{1.25}} \left(\frac{\mu_0 c_{p_0}}{\mu_s c_{p_s}} \right)^{1.25} \tag{1.1}$$

Under the condition that the Prandtl number P = 1 and the Lewis number L = 1, we have the following for the density of the heat flux

[1] Numbers in the margin indicate pagination in the foreign text.

$$q = -\frac{\Lambda}{C_p}\frac{\partial i}{\partial y} \qquad \left(i = \int_0^T c_p dT + i^\circ\right) \tag{1.2}$$

Here i° is the heat of formation of a given component, C_p is the heat conductivity of the gas mixture.

Integrating equation (1.1) through the thickness of the enthalpy boundary layer and introducing the concept of the thickness of the total energy loss

$$\delta_i^{**} = \int_0^{\delta_i} \frac{\rho w}{\rho_0 w_0}\left(\frac{i - i_0}{i_w - i_0}\right) dy \tag{1.3}$$

we produce an integral relationship for the energy of the boundary layer

$$\frac{dR_i^{**}}{dX} + \frac{R_i^{**}}{\Delta i}\frac{d\Delta i}{dX} - \frac{j_w}{\rho_0 w_0} R_L = R_L \frac{q_w}{\rho_0 w_0 \Delta i} \tag{1.4}$$

$$R_i^{**} = \frac{\rho_0 w_0 \delta_i^{**}}{\mu_0}, \quad X = \frac{x}{L}, \quad R_L = \frac{\rho_0 w_0 L}{\mu_0}, \quad \Delta i = i_w - i_0$$

Here j_w is the transverse flow of material at the wall.

Let us analyze the turbulent /104
boundary layer on an impermeable wall beyond the area where the foreign material is injected (Figure 1), when the heat flux through the wall $q_w = 0$. The cooled gas is fed in through a tangential slit (Figure 1a) or an initial porous sector (Figure 1b). If the cooling agent used is a liquid and it evaporates or chemical reactions occur in the initial sector of the plate, the effect of the gaseous products thus formed is equivalent to the injection of a foreign gas.

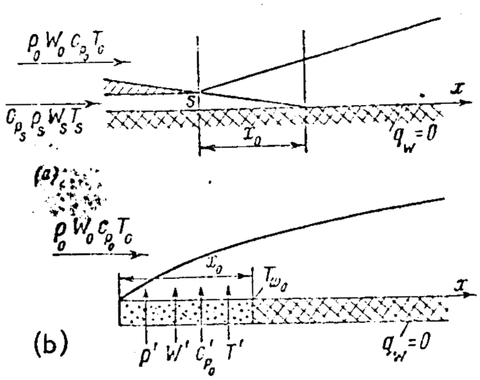

Figure 1

In order to determine the temperature of a heat insulated wall (thermal effectiveness), let us use a method presented in [2, 3].

Integrating equation (1.4) by length from x_0 to x where j = 0 and $q_w = 0$, we produce

$$R_i^{**}\Delta i = R_{i_0}^{**}\Delta i_0, \quad \theta_i = \frac{i_0 - i_w^*}{i_0 - i_{w_0}} = \frac{R_{i_0}^{**}}{R_i^{**}} = \frac{\delta_{i_0}^{**}}{\delta_i^{**}} \qquad (1.5)$$

$$(\theta_i = 1 \text{ where } 0 < x < x_0)$$

Here i^*_w is the total enthalpy of the mixture of gases on the heat insulated wall; i_{w_0} and $\delta^{**}_{i_0}$ are the values of total enthalpy of the mixture of gases on the wall and the thickness of total energy loss in the cross section $x = x_0$.

The maximum intensity of turbulent mixing occurs in the area of the wall (but outside the viscous sublayer) where $\partial w_x/\partial y \to \max$. Therefore, in the boundary layer area near the wall, equilibration of the flow parameters will occur more rapidly and, as in works [2, 3], where $x \to \infty$ we can write the following for the quasi-isothermal flow with exponential approximation of the velocity profile (n = 1/7)

$$\beta = \frac{\delta_i^{**}}{\delta^{**}} \to \beta_{\max} = 9 \quad \left(\delta^{**} = \int_0^\delta \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right) dy\right) \qquad (1.6)$$

here $\rho \to \rho_w \to \rho_0$, δ^{**} is the thickness of loss of momentum in the boundary layer.

The thickness of loss of momentum can be found by solving the equation for momentum for a flat plate [3], and where $x \to \infty$ for quasi-isothermal flow of an incompressible fluid

$$R^{**} = [A(m+1)R_x]^{1/(m+1)} \quad (R^{**} = \rho_0 w_0 \delta^{**}/\mu_0) \qquad (1.7)$$

Here A and m are the coefficient and exponent in the exponential approximation of the friction rule (A = 0.0128, m = 0.25 for a stepped profile with n = 1/7).

Using equations (1.5)-(1.7), we can construct an interpolation formula

$$\theta_i = [1 + 0.24 R_{\Delta x}/R_{i_0}^{**1.25}]^{-0.8} \quad (1.8)$$

For the case of supply of coolant through an initial porous sector in the presence of chemical reactions or phase transformations in the initial sector of the wall, the value of the Reynolds number $R^{**}_{i_0}$, constructed through the thickness of total energy loss, can be found from the solution of the energy equation in the initial sector.

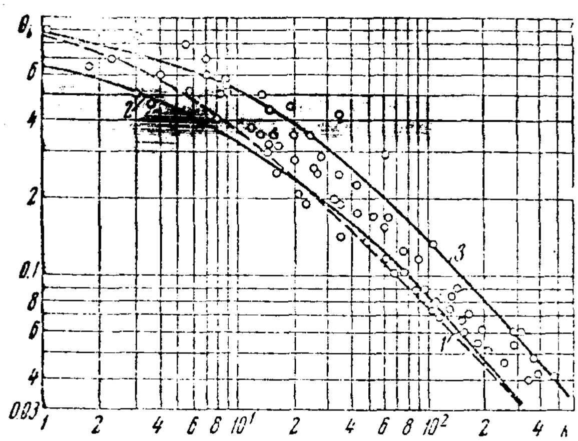

Figure 2

When the gas is blown in through a tangential slit [4]:

$$R_{i_0}^{**} = \frac{\rho_s w_s s}{\mu_0} = R_s \frac{\mu_s}{\mu_0} \quad (1.9)$$

We can see from expression (1.8) that the thermal effectiveness, written through the total enthalpy during injection of a foreign gas, is described by the same formula as when the homogeneous gas is injected. If the principle of superposition of thermal fields is used for solution, we can produce formulas similar to those produced earlier /105 in work [4]:

$$\theta_i = \left\{\left[1 + \frac{62.5}{k + 0.143}\right]^{0.114} - 1\right\}^{0.8} [1 + 0.016k]^{-0.16} \text{ where } \frac{w_s}{w_0} \leqslant 1 \quad (1.10)$$

$$\theta_i = \left\{\left[1 + \frac{62.5}{k + 2}\right]^{0.2} - 1\right\}^{0.8} [1 + 0.016k]^{-0.16} \text{ where } \frac{w_s}{w_0} \approx 1 \quad (1.11)$$

$$\left(k = \frac{R_{\Delta x}}{R_s^{1.25}} \left(\frac{\mu_0}{\mu_s}\right)^{1.25}\right).$$

Curves 1, 2 and 3 on Figure 2 were produced by calculation using formulas (1.8), (1.10) and (1.11) respectively when helium was blown through a tangential slit into the air stream; the points are the results of the experiments of Papell and Hatch [1]. The calculated material agrees satisfactorily with the experimental data. In practical calculations, it is necessary to determine the wall temperature, for which we must know the heat capacity of the mixture of gases on the wall.

The mass transfer equation, without considering thermo- and barodiffusion, has the form

$$\rho w_x \frac{\partial K}{\partial x} + \rho w_y \frac{\partial K}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial K}{\partial y}\right) \tag{1.12}$$

Here K is the total concentration of the component injected. It follows from equations (1.1) and (1.12) that there is similarity between the fields of total enthalpies and weight concentrations with similar boundary conditions.

In this case we have

$$\theta_i = \frac{i_0 - i_w^*}{i_0 - i_{w_0}} = \frac{K_0 - K_w^*}{K_0 - K_{w_0}}, \quad \text{or} \quad K_w^* = K_0 - \theta_i (K_0 - K_{w_0}) \tag{1.13}$$

Here K^*_w is the concentration of the component injected on the wall in the cross section in question; K_{w_0} is the concentration of the component injected on the wall in cross section $x = x_0$. The heat capacity of the mixture of gases on the wall is

$$c_{pw}^* = c_{p_s} K_w^* + c_{p_0}(1 - K_w^*) = c_{p_0} + (c_{p_s} - c_{p_0}) K_w^* \tag{1.14}$$

From equation (1.14) and the expression for θ_i

$$\theta_i = \frac{c_{p_w}^* T_w^* - c_{p_0} T_0}{c_{p_{w_0}} T_{w_0} - c_{p_0} T_0} \tag{1.15}$$

we produce the relationship

$$\theta_t = \frac{T_w^* - T_0}{T_{w_0} - T_0} = \frac{\theta_i (c_{p_{w_0}} T_{w_0} - c_{p_0} T_0) - (c_{p_{w_0}} - c_{p_0}) T_0 K_w^*}{[c_{p_0} + (c_{p_{w_0}} - c_{p_0}) K_w^*] (T_{w_0} - T_0)} \tag{1.16}$$

Here K^*_w is found from (1.13). In the case of injection of a foreign gas through a tangential slit

$$K_0 = 0, \quad K_{w_0} = 1; \quad T_{w_0} = T_s, \quad c_{p_{w_0}} = c_{p_s}$$

from equations (1.13) and (1.16) we produce the dependence

$$\theta_t = \frac{\theta_i c_{p_s}}{\theta_i (c_{p_s} - c_{p_0}) + c_{p_0}} \tag{1.17}$$

This formula corresponds with the formula produced earlier for this case in [1].

2. Suppose the energy loss thickness is represented, as in [3], by the expression

$$\delta_T^{**} = \int_0^{\delta_T} \frac{c_p \rho w}{c_{p_0} \rho_0 w_0} \left(1 - \frac{T - T_w}{T_0 - T_w}\right) dy \tag{2.1}$$

Then the integral relationship for the energy is written through the temperature [3]

$$\frac{d\delta_T^{**}}{dx} + \frac{\delta_T^{**}}{\Delta T} \frac{d(\Delta T)}{dx} - \frac{j_w c_p'}{\rho_0 w_0 c_{p_0}} = \frac{q_w}{c_{p_0} \rho_0 w_0 \Delta T} \tag{2.2}$$

This equation is correct only in the case when the heat capacities of the main gas and the injected gas do not differ strongly. Then, performing the same operations with (2.2) that we perform with (1.4), we can produce formulas for θ_t similar to (1.8), (1.10) and (1.11), with the sole difference that in place of $R^{**}_{i_0}$ we must substitute /106

$$R_T^{**} = R_s \frac{c_{p_s}}{c_{p_0}} \frac{\mu_s}{\mu_0} \tag{2.3}$$

and correspondingly

(2.4)

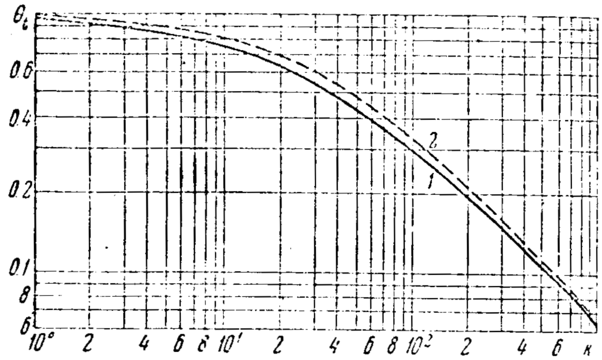

Figure 3

Figure 3 shows a comparison of calculations performed through the enthalpy according to formula (1.8) and formula (1.9) (curve 1), as well as calculations performed according to formula (1.8) considering (2.3) (curve 2), for the case of injection of helium into air. As we see, even for the case when $c_{p_s}/c_{p_0} = 5.2$, the calculations differ quite little.

Figure 4 shows a comparison of calculation for θ_t (curves 1, 2 and 3 respectively) according to formulas (1.10), (1.11) and (1.8) considering (2.3) and (2.4) with experiments on slit cooling by injection at $0 < w_s/w_o < 1$ performed by various authors: points 4 (dark round points) represent helium injected into air [1], points 5 represent air injected into air $T_s/T_o \approx 0.6$ [1], points 6 represent air injected into air $T_s/T_0 \approx 0.3$ [5], and points 7 represent air injected into air $T_s/T_0 \approx 1$ [6].

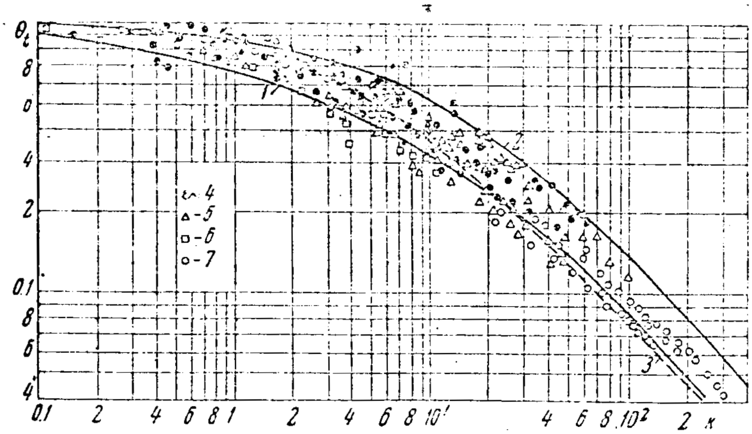

Figure 4

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